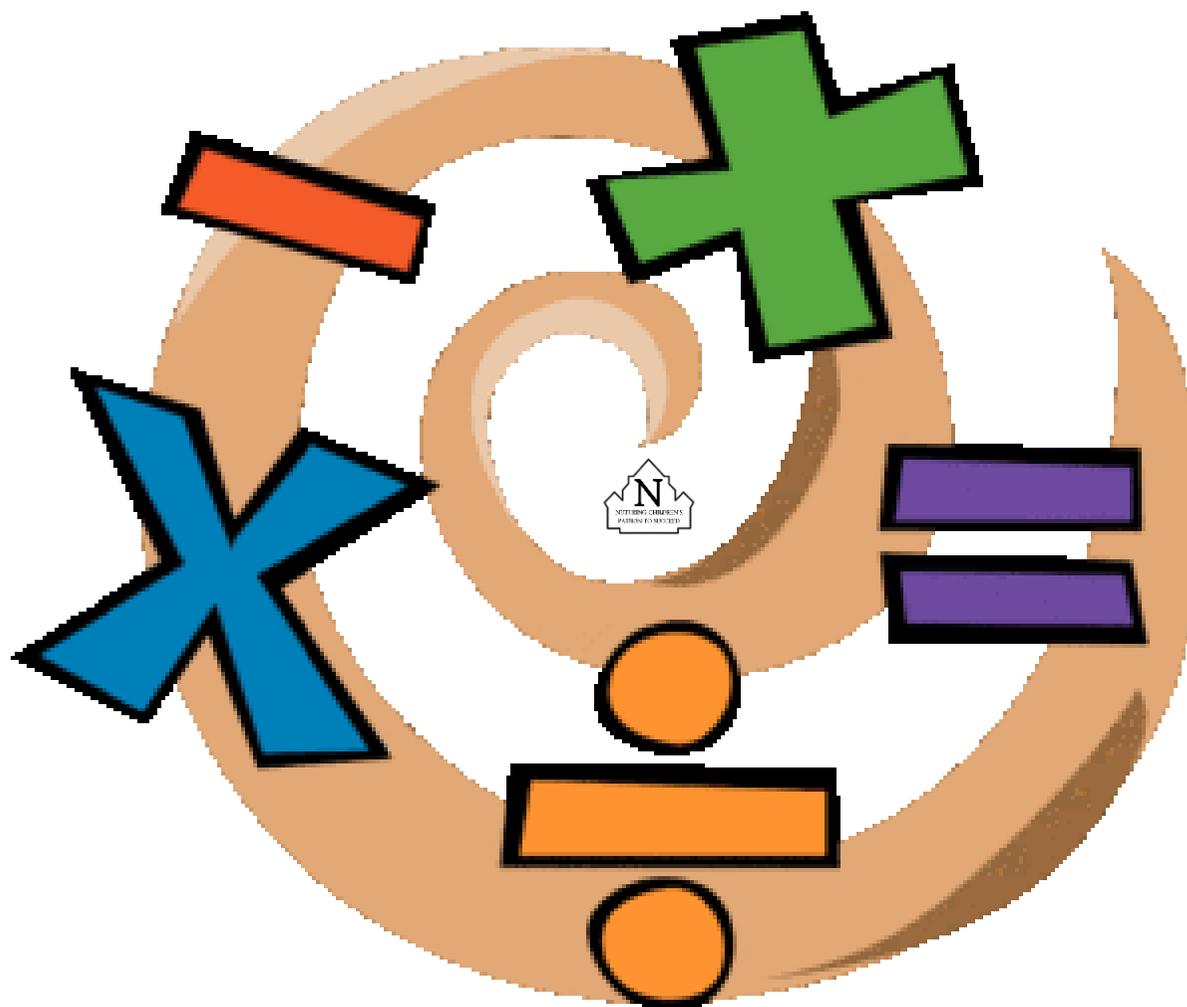




Maths Calculation Overview

*Nanpean Community Primary School
Nurturing Children's Passion to Succeed*

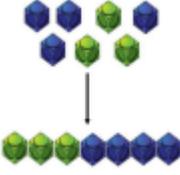
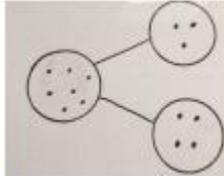
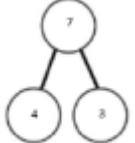
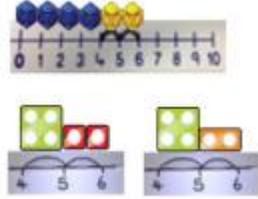
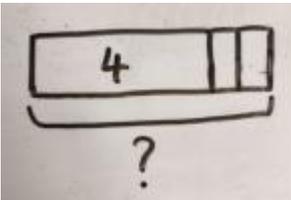


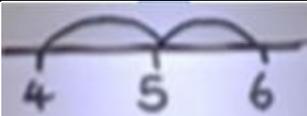
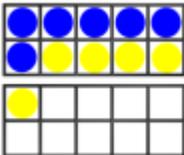
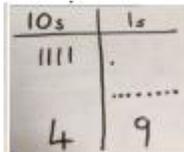
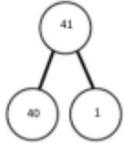
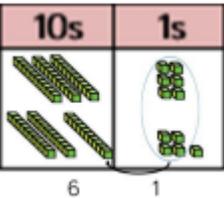
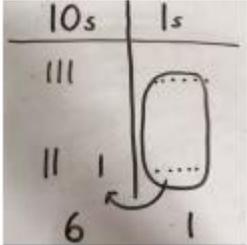
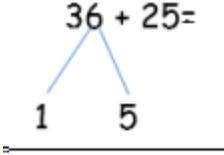
Progression in Addition

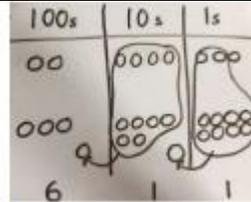
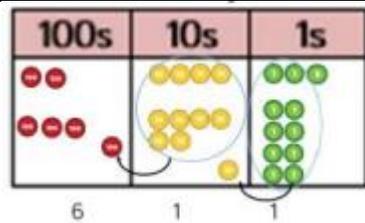
Key Vocabulary: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as', addend + addend = sum

EYES

Before addition can be introduced, children need to have a secure knowledge of number in order to begin addition. To do this, children need to become familiar with all numbers to 20 and understand what those numbers mean. Children are then introduced to the concept of addition through practical games and activities. This is reinforced by opportunities provided in the outdoor area for the children to use addition e.g. adding together groups of building blocks, twigs etc. Children build on their previous knowledge of 'more' by learning that adding two groups of objects together gives them a larger number (more objects). Adults model addition vocabulary supported by age appropriate definition. An example of this is "addition means we add two groups together / we put 2 lots of objects together. Equals means we find out how many we have got altogether. 3 add 2 equals 5! We have got 5 altogether". Adults support children in recording their addition calculations in the written form on whiteboards and in their maths book

<i>Strategy</i>	<i>Concrete</i>	<i>Pictorial</i>	<i>Abstract</i>
Combining two parts to make a whole	<p>Use other resources too e.g. eggs, shells, teddy bears, cars)</p>  <p>Use real life relevant examples for children.</p>	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p> 	<p>$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven</p> 
Counting on using number lines	<p>Using cubes or Numicon first, then alongside a number line. Reknreks can also be used</p> 	<p>A bar model which encourages the children to count on, rather than count all.</p> 	<p>The abstract number line:</p> <p>What is 2 more than 4?</p> <p>What is the sum of 2 and 4?</p> <p>What is the total of 4 and 2?</p> <p style="text-align: center;">$4 + 2$</p>

			
Regrouping to make 10	Using ten frames and counters/cubes, Numicon or Reknreks. $6 + 5$ 	Children to draw the ten frame and counters/cubes 	Children to develop an understanding of equality e.g. $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$
TO + Os Place value counters and bead stings can also be used.	Using base 10. Continue to develop understanding of partitioning and place value. $41 + 8$ 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	$41 + 8$ $1 + 8 = 9$ $40 + 9 = 49$ 
TO + TO with exchanging	Using base 10. Continue to develop understanding of partitioning and place value. $36 + 25$ 	Children to represent the base 10 in a place value chart 	Looking for ways to make 10. (Bridging 10) $30 + 20 = 50$ $5 + 5 = 10$ $50 + 10 + 1 = 61$ 
Use of place value counters to add HTO + TO, HTO + HTO etc	When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred	Children to represent the counters in a place value chart, circling when they make an exchange	Formal written method – column addition



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ 11 \end{array}$$

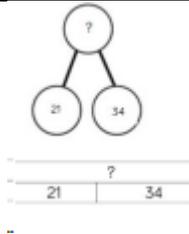
Adding up to 5-digit numbers

Recap above emphasising the 'exchange' process

Recap above emphasising the 'exchange' process.

Children should be confident in using the formal written method of column addition. Focus on procedural variation – making links between calculations.

Conceptual Variation different ways to ask children to solve $21 + 34 =$



Word problems: In year 3, there are 21 children and in year 4, there are 34 children. How many children in total? $21 + 34 = 55$. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$21 + 34 = \underline{\quad}$

$\underline{\quad} = 21 + 34$

Calculate the sum of twenty-one and thirty-four.

10s	1s

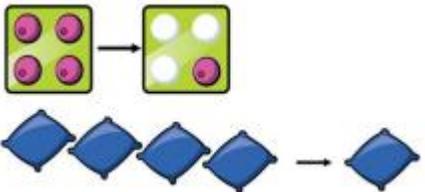
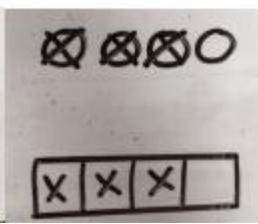
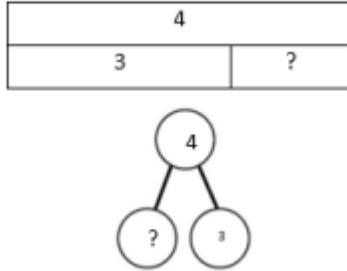
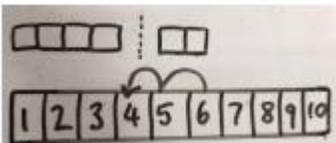
Missing digit problems

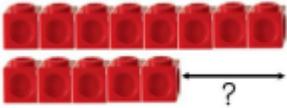
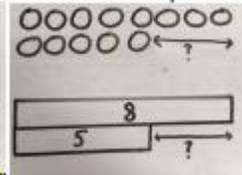
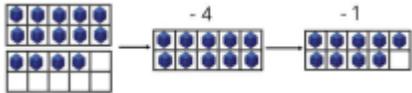
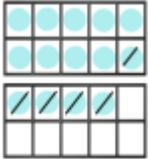
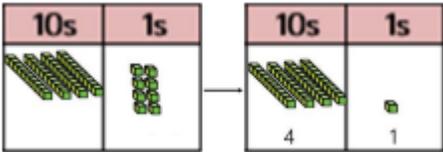
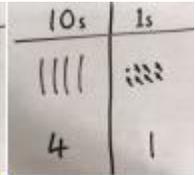
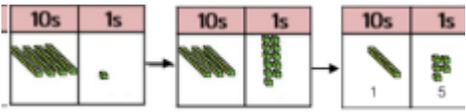
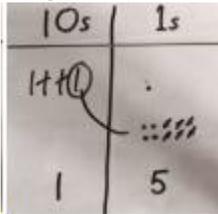
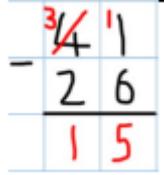
Progression in Subtraction

Key Vocabulary: take away, less than, the difference, subtract, minus, fewer, decrease, difference

EYES

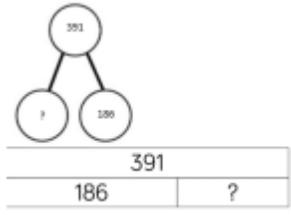
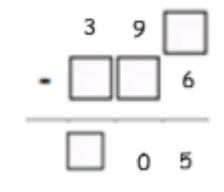
Before subtraction can be introduced, children need to have a secure knowledge of number in order to begin subtraction. Children are then introduced to the concept of subtraction through practical games and activities. This is reinforced by opportunities provided in the outdoor area for the children to count e.g. counting building blocks, twigs etc. Children build on their previous knowledge of 'less' by learning that subtracting means taking away a certain number of objects from a group (leaving them with less objects). Adults model subtraction vocabulary supported by age appropriate definition. An example of this is "subtraction means we take away objects from a group / we have 11 got less objects now. Equals means we find out how many we have got left. Wow! We have only got 3 left!" Adults support children in recording their subtractions in the written form on whiteboards and in their maths books

<i>Strategy</i>	<i>Concrete</i>	<i>Pictorial</i>	<i>Abstract</i>
Physically taking away and removing objects from a whole	<p>Ten frames, Numicon, cubes and other items such as beanbags could be used.</p> <p style="text-align: center;">$4 - 3 = 1$</p> 	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	<p>$4 - 3 = \underline{\quad}$</p> <p>$\underline{\quad} = 4 - 3$</p> 
Counting back	<p>Using number lines or number tracks Children start with 6 and count back 2. $6 - 2 = 4$</p> 	<p>Children to represent what they see pictorially e.g.</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line.</p> 
Finding the difference	<p>Using cubes, Numicon or Reknreks, other objects can also be used. Calculate the difference between 8 and</p>	<p>Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what</p>	<p>Find the difference between 8 and 5.</p>

	<p>5. Children to draw the cubes/other concrete objects which they have used or use the bar</p> 	<p>they need to calculate</p> 	<p>8 – 5, the difference is ____</p> <p>Children to explore why $9 - 6 = 8 - 5 = 7 - 4$ have the same difference</p>
<p>Making 10</p>	<p>Using ten frames. 14 – 5</p> 	<p>Children to present the ten frame pictorially and discuss what they did to make 10</p> 	<p>Children to show how they can make 10 by partitioning the subtrahend.</p> $14 - 5 = 9$ $\begin{array}{c} 4 \\ \swarrow \quad \searrow \\ 1 \quad 1 \end{array}$ $14 - 4 = 10$ $10 - 1 = 9$
<p>Using partitioning</p>	<p>Using base 10 or place value counters 48-7</p> 	<p>Children to represent the base 10 pictorially</p> 	<p>Children to write out their mental strategy.</p> $48 - 7 =$ $40 - 0 = 40$ $8 - 7 = 1$ $40 + 1 = 41$
<p>Using partitioning with an exchange – leading to column subtraction.</p>	<p>Using base 10 or PV counters and having to exchange. 41 – 26</p> 	<p>Represent the base 10 pictorially, remembering to show the exchange</p> 	<p>Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$</p> 
<p>Subtracting up to 7-digit numbers</p>	<p>Recap above emphasising the</p>	<p>Recap above emphasising the</p>	<p>Children should be confident in using the</p>

using column subtraction.	'exchange' process.	'exchange' process	formal written method of column subtraction. Focus on conceptual variation and procedural variation.
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Conceptual Variation different ways to ask children to solve 391 - 186

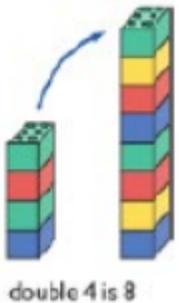
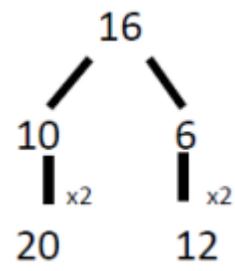
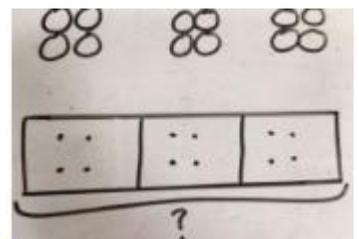
	<p>Word problems Raj spent £391, Timmy spent £186. How much more did Raj spend? Calculate the difference between 391 and 186.</p>	<p><u> </u> = 391 - 186</p> <p>391 What is 186 less than 391?</p> <p><u>-186</u></p> <p>_____</p>	
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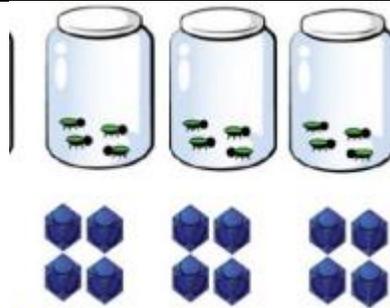
Progression in Multiplication

Key Vocabulary: double, times, multiplied by, the product of, groups of, lots of, equal groups, exchange. Multiplicand x multiplier = product

EYES

By the end of EYFS, children are expected to understand the concept of doubling and to be able to double a number up to 10. Before doubling can be introduced, children need to have a secure knowledge of counting, number facts and addition in order to double. Children are then introduced to the concept of doubling through practical games and activities, including the use of the outdoor areas. Children act out 'doubling' by physically add two equal groups together to find out the 'doubles' answer

<i>Strategy</i>	<i>Concrete</i>	<i>Pictorial</i>	<i>Abstract</i>
Doubling	<p>Use practical activities to show how to double a number</p> 	<p>Draw pictures to show how to double a number</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together</p>
Repeated grouping / Repeated addition	<p>Use different objects to add equal groups</p>  <p>'There are 3 equal groups with 3 in each' $3 + 3 + 3 = 9$</p>	<p>Children to represent the practical resources in a picture and use a bar model.</p>  <p>There are 3 plates, each plate has 2 apples. How many</p>	<p>Write addition calculations to describe pictures and objects.</p>  <p>$2 + 2 + 2 = 6$</p>

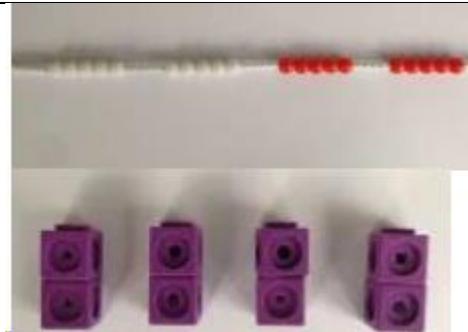


'There are 3 equal groups, with 4 in each.' $4 + 4 + 4 = 12$

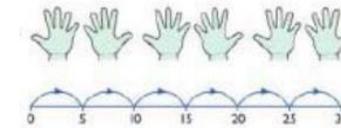
apples are there all together?



Counting in multiples



Count in multiples supported by concrete objects in equal groups.



Use a number line or pictures to continue support in counting in multiples

Count in multiples of a number aloud.
Write sequences with multiples of numbers.

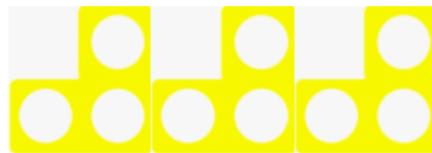
2, 4, 6, 8, 10

5, 10, 15, 20, 25, 30

Start from different numbers to count in multiples, **not just zero.**

Number lines to show repeated groups

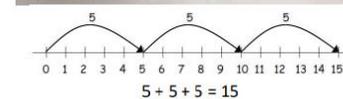
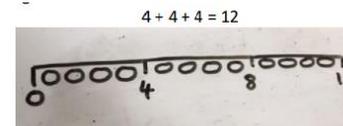
Numicon



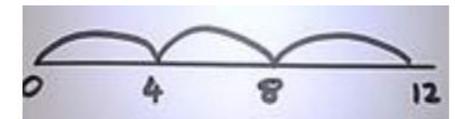
Use a ruler to show a number line

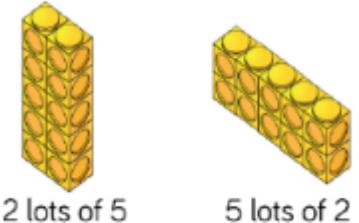
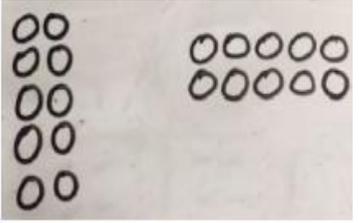
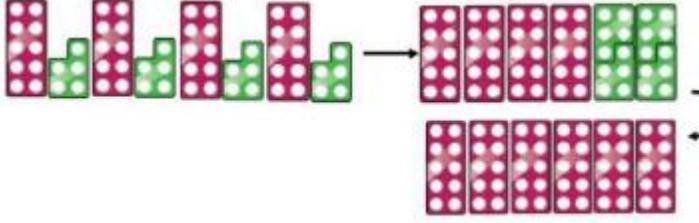
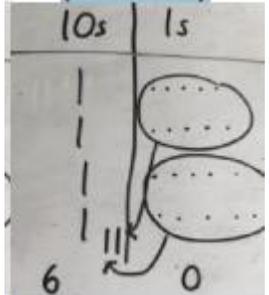
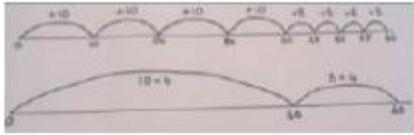


Represent this pictorially alongside a number line e.g.

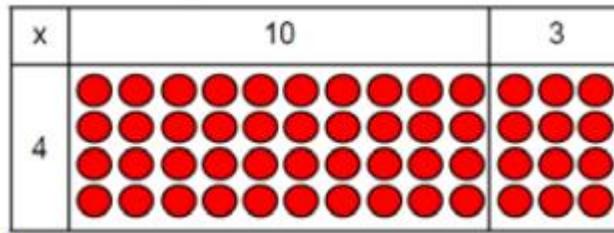


Abstract number line showing three jumps of four. $3 \times 4 = 12$



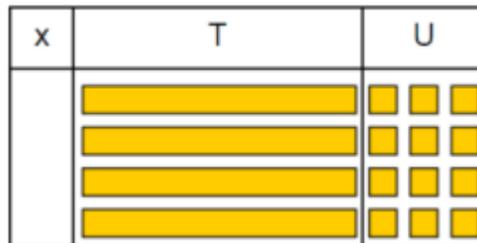
<p>Use arrays to illustrate commutativity</p>	<p>Counters and other objects can also be used. $2 \times 5 = 5 \times 2$</p> 	<p>Children to represent the arrays pictorially.</p>  <p>Link arrays to area of rectangles (Y4)</p>	<p>Children to be able to use an array to write a range of calculations e.g.</p> $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$
<p>Partition to multiply</p>	<p>Using Numicon or base 10 (Dienes)</p> <p>$4 \times 15 =$</p> 	<p>Children to represent the concrete manipulatives pictorially</p>  <p>Clearly show the 'exchange' of the 20 ones for 1 ten.</p>	<p>Children to be encouraged to show the steps they have taken</p> 4×15 $\begin{array}{r} 10 \quad 5 \\ 10 \times 4 = 40 \\ 5 \times 4 = 20 \\ 40 + 20 = 60 \end{array}$ <p>A number line can also be used</p> 
<p>Grid Method</p>	<p>Show the link with arrays to first introduce the grid method.</p> <p>4 rows of 10</p>	<p>They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as</p>	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p>

4 rows of 3

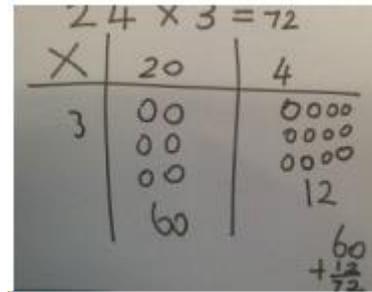


Move on to using Base 10 to move towards a more compact method

4 rows of 13



shown below



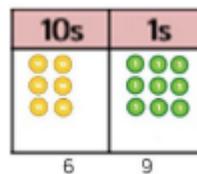
x	30	5
7	210	35

$$210 + 35 = 245$$

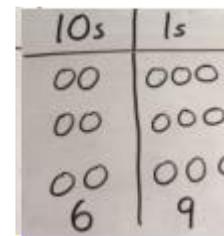
Formal column method no exchanging. 1-digit x 2-digit (Y3)
1-digit x 3-digit (Y4) 1-digit x 4-digit (Y5)

Place value counters can be used to represent (base 10 can also be used.)

$$3 \times 23$$



Children to represent the counters pictorially.



Children to record what it is they are doing to show understanding

$$3 \times 23 \quad 3 \times 20 = 60$$

$$20 \quad 3 \quad 3 \times 3 = 9$$

$$60 + 9 = 69$$

Then they move onto the formal written layout

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

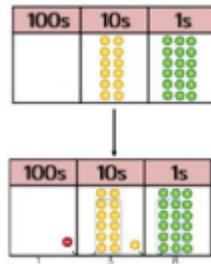
Formal column method with exchanging. 1-digit x 2-digit (Y3)
1-digit x 3-digit (Y4) 1-digit x 4-

Place value counters or Base 10 can be used to represent 6×23

Children to represent the counters/base 10, pictorially

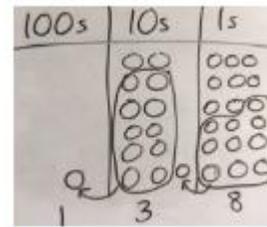
Formal written method. Begin with expanded before moving to compact

digit (Y5)

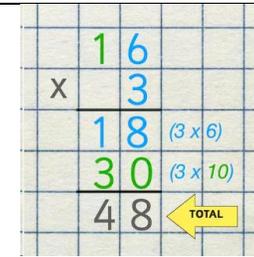


Children use stem sentences to explain that they need to 'exchange 10 ones for 1 ten. And then 10 tens for 1 hundred.'

e.g. the image below



Children to be exposed to the formal written method alongside the pictorial.

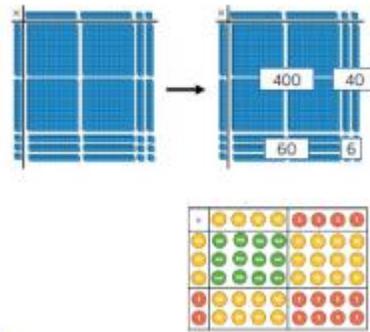


$$6 \times 23 =$$

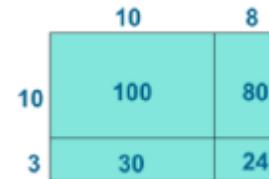
$$\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ 11 \end{array}$$

Formal column method (Long multiplication) 2-digit x 2-digit.

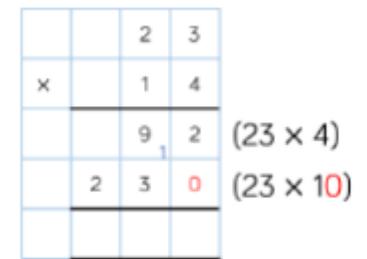
Represent using Base 10 and Place value counters using the grid method.



Moving forward, multiply by a 2 digit number showing the different rows within the grid method.



Formal written method of long multiplication.



Formal written method

When children start to multiply 3d x 3d and 4d x 2d etc., they should be confident with the abstract

When children start to multiply 3d x 3d and 4d x 2d etc., they should be confident with the abstract The grid method can be used as a pictorial representation

Formal written method of long multiplication.

		1	3	2	
x			1	4	
		5	2	8	(132 × 4)
		1	3	2	0 (132 × 10)

Multiplying decimals up to 2 decimal places by a single digit.

Place Value counters can be drawn if necessary.

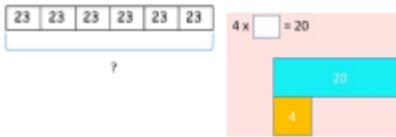
Place Value counters can be drawn if necessary.

Remind children that the single digit belongs in the ones column. Line up the decimal points in the question and the answer.

		3	.	1	9
x		8			
		2	5	.	5
				2	

Conceptual Variation different ways to ask children to solve 6×23

Bar model



Word problems

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? With the counters, prove that $6 \times 23 = 138$

Find the product of 6 and 23

$$6 \times 23 =$$

$$\underline{\quad} = 6 \times 23$$

$$\begin{array}{r} 6 \quad 23 \\ \times \underline{23} \quad \times \underline{6} \\ \hline \quad \quad \end{array}$$

What is the calculation?

What is the product?

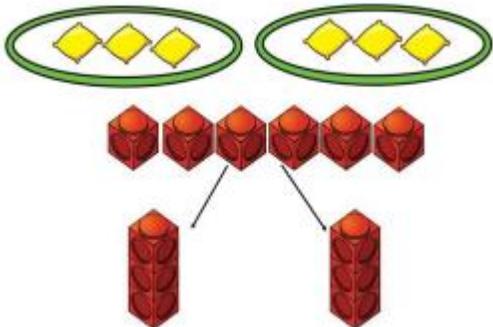
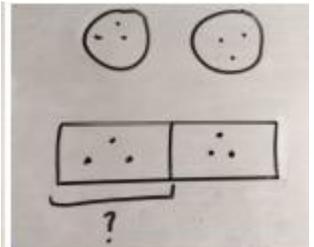
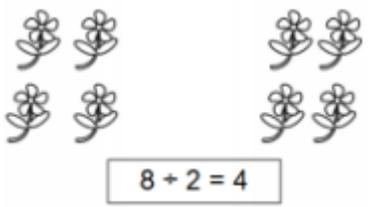


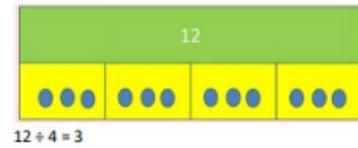
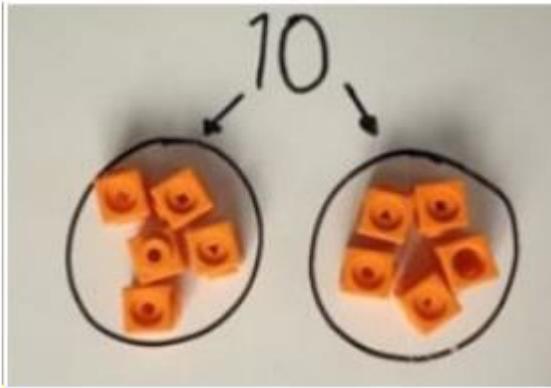
Progression in Division

Key Vocabulary: share, group, divide, divided by, half, repeated subtraction $\text{Dividend} \div \text{divisor} = \text{quotient}$

EYES

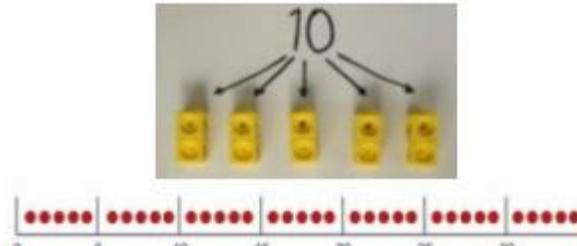
By the end of EYFS, children are expected to understand the concept of halving and sharing. Before this can be introduced, children need to have a secure knowledge of counting backwards, number facts and subtraction in order to halve and share. Children are then introduced to the concept of halving and sharing through practical games and activities. They act out 'halving and sharing' through activities. This is reinforced by opportunities provided in the outdoor area for the children to halve and share out objects such as building blocks, twigs etc.

<i>Strategy</i>	<i>Concrete</i>	<i>Pictorial</i>	<i>Abstract</i>
Halving	<p>Using a range of objects</p> 	<p>Represent the sharing pictorially</p> 	<p>$6 \div 2 = 3$</p> <p>Children should also be encouraged to use their 2 times table facts.</p>
Sharing	<p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities</p>  <p>Children use bar modelling to show and support understanding</p>	<p>$12 \div 3 = 4$</p>

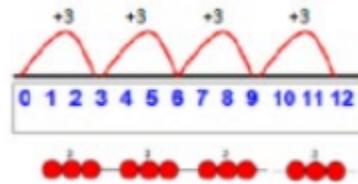


Grouping

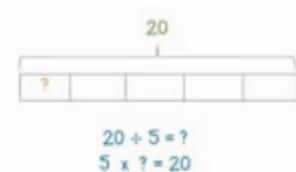
Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.



Use number lines for grouping



Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group



$$28 \div 7 = 4$$

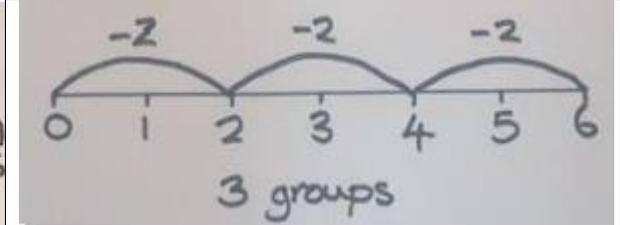
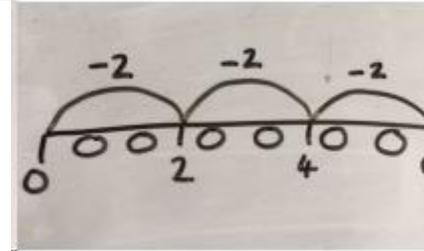
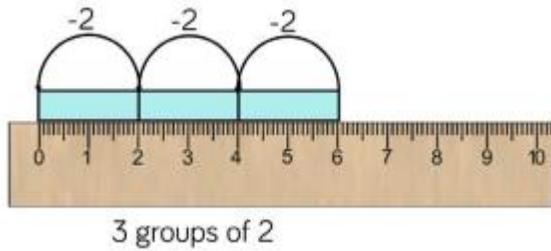
Divide 28 into 7 groups.
How many are in each group?

Repeated subtraction

Using cubes above a ruler. $6 \div 2$

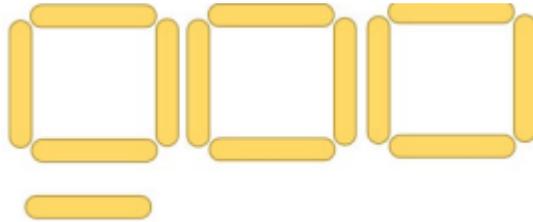
Children to represent repeated subtraction pictorially

Abstract number line to represent the equal groups that have been subtracted



2d ÷ 1d with remainders

Using lollipop sticks. Cuisenaire rods, above a ruler can also be used. $13 \div 4$ Use of lollipop sticks to form wholes- squares are made because we are dividing by 4



There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.

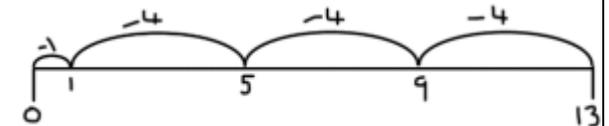


There are 3 whole squares, with 1 left over

$13 \div 4 = 3$ remainder 1

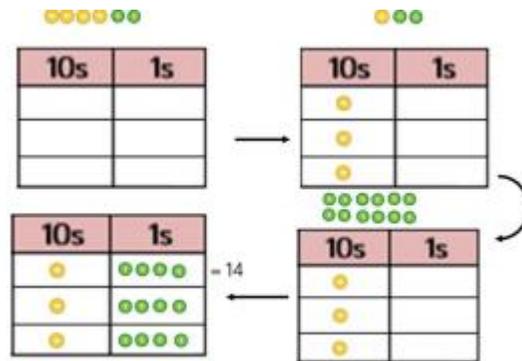
Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

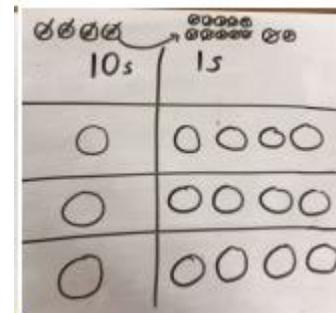


Sharing using place value counters

$$42 \div 3 = 14$$



Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

$$42 \div 3$$

$$42 = 30 + 12$$

$$30 \div 3 = 10$$

$$12 \div 3 = 4$$

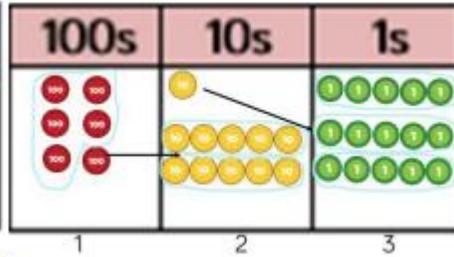
$$10 + 4 = 14$$

Short division

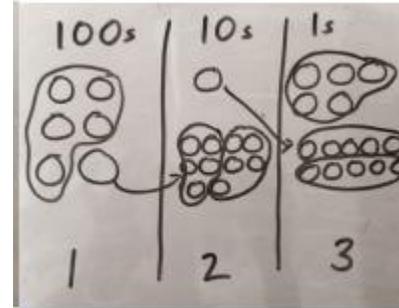
Using place value counters to group. $615 \div 5$

Represent the place value counters pictorially

Children to the calculation using the short division scaffold. (Division bracket/Bus Stop)



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?



$$5 \overline{) 615}$$

$\overset{1}{6}$
 $\overset{1}{1}$
 5

Long division

$$354 \div 15 = ?$$

$$\begin{array}{r} 23 \text{ r}9 \\ 15 \overline{)354} \\ \underline{30} \\ 54 \\ \underline{45} \\ 9 \end{array}$$

Multiples of 15:
15 30 45 60 75 90 105 120

So, $354 \div 15 = 23 \text{ r}9$

$$\begin{array}{r} 23 \frac{9}{15} \\ 15 \overline{)354} \\ \underline{30} \\ 54 \\ \underline{45} \\ 9 \end{array}$$

Multiples of 15:
15 30 45 60 75 90 105 120

$$\frac{9}{15} = \frac{3}{5}$$

So, $354 \div 15 = 23 \frac{3}{5}$

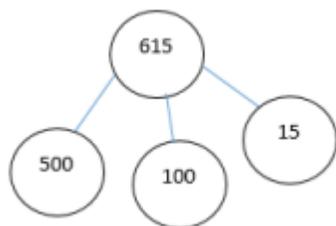
$$\begin{array}{r} 23.6 \\ 15 \overline{)354.0} \\ \underline{30} \\ 54 \\ \underline{45} \\ 90 \\ \underline{90} \\ 0 \end{array}$$

Multiples of 15:
15 30 45 60 75 90 105 120

So, $354 \div 15 = 23.6$

Conceptual Variation different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\underline{\quad} = 615 \div 5$$

What is the calculation?

What is the quotient?

